HYDRAULIC RELIABILITY OF PRESSURIZED WATER DISTRIBUTION NETWORKS FOR ON-DEMAND IRRIGATION

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Abstract

A new probabilistic approach is proposed to correctly evaluate the performance of pressurized water distribution networks for on-demand irrigation, which is based on the preliminary generation, by the Monte Carlo Method, of a number N of equally-probable water demand scenarios, and on the subsequent analysis, by means of a proper hydraulic model, of the actual behaviour of the network in each of the scenarios. Four different performance indexes are then proposed, allowing the evaluation of the system reliability, with reference to the capability of satisfying the users' demand, both at local and at global level. A case-study is also examined to provide guide-lines for correct application of the proposed approach.

Keywords
Irrigation, reliability, performance indexes, hydraulic networks, water distribution, performance analysis

1. INTRODUCTION

In contrast with the case of traditional irrigation systems, pressurized networks allow the farmers to take water from distribution network when and how desired. Since the number M_{spr} and the position of hydrants, actually operating at a given time, are unknown, the design of pressurized irrigation networks, consisting of $n_{spr.}$ hydrants, is accomplished with the adoption of traditional probabilistic approaches ([1], [2]). In contrast with the apparent adequacy of the probabilistic approach, and depending on the characteristics of the area to serve and the water distribution system (topography, diameter and roughness of the pipes, topology, etc.), the actual availability of the discharges at the hydrants can be quite different from what is expected, since the hydraulic heads at the nodes depend on the random number and position of the simultaneously operating hydrants. In this paper, the topic of hydraulic reliability of pressurized irrigation networks is addressed, with special regard to the hydraulic reliability of rotating sprinklers irrigation networks. Given a water distribution network, a number of demand patterns (number and position of operating hydrants) are generated by means of the Monte Carlo Method, and for each situation the hydraulic behaviour of the network is evaluated. For each operating condition, local Performance Indexes (referred to the nodes where a hydrant is present) and global Performance Indexes (referred to the whole distribution network) are firstly defined and then evaluated. Once the Probability Distribution Functions of the Performance Indexes are estimated, the Reliability Functions can be evaluated. A numerical example clarifies the proposed approach: the calculations show that, in some cases, the probability that the Performance Indexes attain values smaller than 1 can be well within the not-exceedence probability P used for the probabilistic design.

2. DESCRIPITION OF THE PROPOSED APPROACH

2.1 Generalities

The approach proposed in the present paper, according to well established methodologies, is analogous to that proposed by two of the Authors [3]. It consists of the following steps.
1) Preliminary sizing of the network, with reference to: a) ideal operating conditions, with discharges delivered by the hydrants not depending on the potential heads at nodes; b) a probabilistic approach very similar to that proposed by Clement ([1], [2]); c) a technical-economic criterion of least-cost and maximum safety for the choice of the diameters to be assigned to each branch of the network; 2) Generation, through the Monte Carlo Method, of a number N of possible and equally-probable scenarios of users demand; 3) Hydraulic simulation of the system for each of the N operating conditions produced; 4) Evaluation of local and global Performance

Indexes, for each of the N operating conditions, and estimation of their Cumulative Distribution Functions (*CDF*); 5) Estimation of the local and global *Reliability Functions* of the system, as complement to 1 of the *CDF*s of the Performances Indexes; 6) Evaluation of the *hydraulic reliability* of the system, at local and global level, by estimating the value of the Reliability Function corresponding to the design value of the Performance Indexes.
In the following paragraphs, the main characteristics of the numerical models used for the development of the methodology proposed are depicted.

2.2 Preliminary sizing of the network

The preliminary design of the network is performed using a classical probabilistic approach, firstly proposed by Clement [1] [2]: for a preset non-exceedence probability, it is possible to estimate the maximum discharge flowing in each branch of the distribution network. It is based on the following hypotheses: i) the discharge $q_{spr.}$ distributed by each of the hydrants, is independent from the actual head at the node where the hydrant is located; ii) the hydrants can be regulated only in the positions ON/OFF (i.e., completely open or completely closed), with the same elementary probability p of opening. In such hypotheses, the discharge Q^s flowing through the link s is proportional to the number of hydrants actually open downstream. It's easy to verify that the probability distribution of the number $M^s_{spr.}$ of hydrants downstream the link s which are contemporarily open, over the total number $n^s_{spr.}$, is binomial. Then, given a design value of the failure probability $P_{exc.}$, the maximum number $m^s_{spr.}$ of sprinklers contemporarily open with probability $1-P_{exc.}$ can be easily evaluated. In the case that a number z of different types of hydrants are operated, each with a proper value $q_{spr.}$ of the design discharge distributed, a similar approach applies, recalling that the sum of two binomial variates, with elementary probability p, is a binomial variate too. Once the design discharges in each link of the network are known, economical constraints can be used to evaluate the diameter of the pipes. In most cases the potential heads for network operation have to be supplied by a pumping station, and it is possible to make reference to a simple economic criterion, such as the one proposed by Bresse, or similar, like:

$$D_{theor.} = 1.2 \cdot \left(Q^s\right)^{0.45} \tag{1}$$

where $D_{theor.}$ is the theoretical diameter. The usage of this formula can be extended to all the links of the network.

2.3 Application of the MonteCarlo Method to generate scenarios of users' water demand

In order to generate N different scenarios for the users' demand, the well known Monte Carlo Method can be used. The generation of the random variable M_i, whose distribution is binomial, with elementary probability p, can be accomplished by means of the algorithm outlined in [12]. The general procedure for z different types of hydrants is the following. For each operating condition, $G=z+n$ random variates R, uniformly distributed in the range (0,1), are generated, where n is the total number of sprinklers of the network. The fist z values of R (R_1, R_2, ...R_z) are used to supply the number M_i ($i=1,2,\ldots,z$) of hydrants of the type i contemporarily open. The remaining uniformly distributed n random numbers (R_{z+1}, R_{z+2}, ...R_{z+n}) are assigned each to a different sprinkler of the network (R_{z+1} to the first sprinkler, R_{z+2} to the second sprinkler and so on): finally, for each type of sprinkler, the hydrants with the greatest M_i values of R_{z+j} are selected, and considered open. As a consequence, the procedure adopted in this paper is quite different from the one considered in [8].

2.4 Hydraulic modelling of on-demand irrigation networks

The hydraulic model applied is based on the solution of a set of S non-linear equations with S unknowns (the hydraulic heads at the nodes of the water distribution system, h_j) [3] [13]. The set of equations is obtained writing, in each node, the continuity equation as a function of the hydraulic head at the same node and the hydraulic heads in the nodes directly linked to it [6] [7]. When the flow is oriented from the node n to the node j (with n upstream of j), the discharge through the pipe connecting the nodes n and j could be evaluated as:

$$Q_{n,j} = \left|\frac{h_n - h_j}{r_{n,j}}\right|^{1/\alpha_{n,j}} \tag{2}$$

In the equation (2), $r = SF \cdot \left(\beta / D^{\omega}\right) \cdot l$, and D and l represent, respectively, the diameter and length of the pipe; α is an exponent assuming values ranging between 1 (laminar flow) and 2 (turbulent flow over rough walls); ω is an exponent assuming values ranging between 4 (laminar flow) and 5 (turbulent flow over rough walls); β is a coefficient that could be itself function of the discharge, the pipe diameter and the roughness of walls, *SF* is a safety factor. On the other hand, the continuity equation at the node *j* could be written as

$$\sum_{n=1}^{N_{j1}} Q_{n,j} - \sum_{z=1}^{N_{j2}} Q_{j,z} - Q_j = 0 \tag{3}$$

where: N_{j1} and N_{j2} = number of pipes respectively inflowing and flowing out the node *j*. Q_j is the discharge delivered by the node *j*, that, generally speaking, depends on the hydraulic head in the node. For instance, if in the node *j* is present a sprinkler, the discharge delivered at that node is given by:

$$Q_j = k_j \cdot \mu_j \cdot \sigma_j \cdot or_j \cdot \sqrt{2g \cdot (h_j - y_j)} = k_j \cdot \lambda_j \cdot (h_j - y_j)^{0.5} \tag{4}$$

where μ_j is the coefficient of discharge, σ_j is the maximum open area of the sprinkler, or_j is the opening ratio of the sprinkler, varying in the range [0,1], given by the ratio between the actual open area and the maximum area; *g* is the gravity acceleration; y_j is the elevation, above the reference horizontal plane, of the barycentre of the opening; and k_j is an index, equal to 0 if the sprinkler is closed and equal to 1 if the sprinkler is open. Combining the equations (2), (3), and (4), for the nodes where a sprinkler is located we can obtain

$$\sum_{n=1}^{N_{j1}} \left[\left| \frac{h_n - h_j}{r_{n,j}} \right|^{1/\alpha_{n,j}} \right] - \sum_{z=1}^{N_{j2}} \left[\left| \frac{h_j - h_z}{r_{j,z}} \right|^{1/\alpha_{j,z}} \right] - k_j \cdot \lambda_j \cdot (h_j - y_j)^{0.5} = 0 \tag{5}$$

Similar equations are obtained for the nodes where pumping stations, oscillation tanks, sluice-gates, check-valves, pressure reduction valves and other devices are present [11]. The resulting set of non-linear equations, where the hydraulic heads h_j in the nodes are unknown, is solved by using to Newton-Raphson method, starting from initial values of potential heads obtained using the approach proposed by Cornish, in the way suggested by Streeter [14]. In order to guarantee the stability of the method in all the circumstances, the Newton-Raphson procedures is applied in the way proposed by Lam and Wolla [6] [7], where the relaxing coefficient is automatically modified from the software until convergence is attained.

2.5 Performance Indexes and Reliability Functions

According to [5], in order to evaluate, for each operating condition, the capability of the system to satisfy the users' demand, proper local and global dimensionless Performance Indexes are introduced. The first of the two Performance Indexes at local level, *LPI_DDN* (*L*ocal *P*erformance *I*ndex related to the *D*ischarges Delivered at a *N*ode), is defined, with reference to the node *j* and the scenario *x* (*x=1,2,…,N*), as

$$(LPI\_DDN)_j^x = \begin{cases} 0 & \text{if} \quad h_{j,x} < y_j \Rightarrow 0 \\ \dfrac{Q_{j,x}}{Q_j^*} = \left(\dfrac{h_{j,x}}{h_{j,opt}} \right)^{0.5} & \text{if} \quad y_j \le h_{j,x} \le h_{j,opt} \\ 1 & \text{if} \quad h_{j,x} > h_{j,opt} \end{cases} \tag{6}$$

where $Q_{j,x}$ is the discharge actually distributed, evaluated with reference to the actual potential head $h_{j,x}$, Q^*_j is the reference discharge to be delivered by the hydrant, evaluated with reference to the optimal potential head, $h_{j,opt}$, and y_j is the topographic elevation of the barycentre of the sprinkler outflow area. It evaluates the level of demand satisfaction at the node *j* with reference to the discharge actually delivered at that node.
The second of the two Performance Indexes at local level, *LPI_CAN* (Local Performance Index related to the Covered Area at a Node), is defined with reference to the node *j* and the scenario *x* (*x=1,2,…,N*) as

$$(LPI\_CAN)_j^x = \begin{cases} 0 & \text{if} \quad h_{j,x} < y_j \Rightarrow 0 \\ \left(\dfrac{h_{j,x}}{h_{j,opt}} \right)^{2} & \text{if} \quad y_j \le h_{j,x} \le h_{j,opt} \\ 1 & \text{if} \quad h_{j,x} > h_{j,opt} \end{cases} \tag{7}$$

It evaluates the level of demand satisfaction at the node *j* with reference to the actually watered plot area: its definition easily follows from the observation that the range *L* of a jet flowing out from a hydrant is proportional to the piezometric head *h* on the hydrant. Interestingly, the indexes defined in the equations (6) and (7) are quite different from the one proposed by Pereira et al. [10], which considers, as performance index, just the ratio between $h_{j,x}$ and $h_{j,opt}$.
The first Performance Indexes at global level, *GPI_DDN* (*G*lobal *P*erformance *I*ndex related to the *D*ischarges *D*elivered at *N*odes), is defined, with reference to the scenario *x* and the M_x nodes actually operating, as:

$$\left(GPI\_DDN\right)^x = \sum_{j=1}^{M_x} Q_{j,x} \Big/ \sum_{j=1}^{M_x} Q_{j,opt} \quad (8)$$

It evaluates the demand satisfaction, at global level, with reference to the discharge delivered by the M_x actually operating nodes. The index defined in the equation (8) is different from the "equality measure" proposed by Pereira et al. [10], but analogous. The second of the two Performance Indexes at global level, *GPI_CA* (Global Performance Index with reference to Covered Area), is defined, with reference to the scenario x ($x=1,2,\ldots,N$) and M_x nodes actually operating, as

$$\left(GPI\_CA\right)^x = \sum_{j=1}^{M_x} \left(h_{j,x}\right)^2 \Big/ \sum_{j=1}^{M_x} \left(h_{j,opt}\right)^2 \quad (9)$$

It evaluates, at global level, the satisfaction of users' water demand with reference to the actually watered plot area by means the M_x operating nodes. In the case of the irrigation systems, the reliability evaluates the ability of the network to satisfy the users' demand. Then, an objective estimation of the Reliability Function could be the probability of exceedence of a certain value of Performance Index, taken as a threshold. Accordingly, after re-arranging of the Performance Indexes in ascending order, the Cumulative Distribution Function are estimated by

$$F_i = P\left[V \le v_i\right] \cong \frac{i-0.5}{N} \quad (10)$$

Then, for each of the Performance Indexes, both local and global, the corresponding *Reliability Function* is evaluated as the complement to 1 of the *CDF* estimations given by the eq. (10).

3. APPLICATION EXAMPLE

3.1 Description of the irrigation network

The approach proposed in the present paper was applied to a schematic study-case (Fig.1): a pressurized network for on-demand irrigation is characterised by 4 main reaches, 152 links, 153 nodes (115 of which could distribute water on-demand). The geometric characteristics of the network are reported in [4].

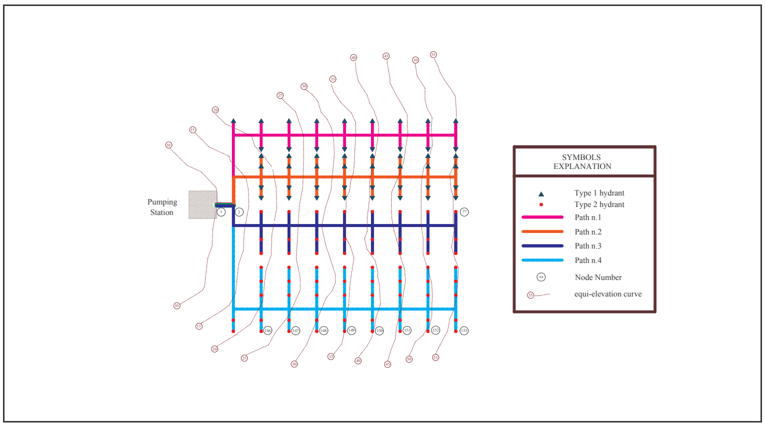

Fig.1. On-demand irrigation network considered in the case-study

The network is served by a pumping station. Two different materials are used for pipes: PVC for diameters up to 350 mm; iron for greater diameters. The parameters to be used in resistance formulas are the following. PVC: β = 0.000775, α = 1.75, ω = 4.75, *SF* = 1.1. IRON: β = 0.00141, α = 1.82, ω = 4.81, *SF* = 1.3. Two different types of hydrants are used for delivering the water to the end-users. The first type (49 over 115) distributes $Q_{j,opt}$ =10 l/s under a potential head h_{jopt} = 60 m, while the second type (66 over 115) supplies $Q_{j,opt}$ = 20 l/s with h_{jopt} = 60 m.

3.2 Network sizing

Two different reference situations were taken in account for the design of the network: a) sizing of the pipes with reference to 5% exceedence probability (Case 1); sizing of the pipes with reference to a 2.5% exceedence probability (Case 2). In both cases p=1/3 was chosen for the elementary probability of hydrants opening. In order to evaluate the system reliability, the same N=1000 demand patterns for both the cases were considered.

Since the small difference of the design discharges in the upstream link for the two cases ($Q_{0.950}$=724,00 l/s and to $Q_{0,975}$=739.74 l/s), the characteristic curve adopted for the pumping station was the same in both cases. Passing from the Case 1 to the Case 2,and considering the eq. (1), only 12 links over 156 exhibited a different diameter .

3.3 Network simulation and evaluation of Reliability Functions

Once generated the demand scenarios, the mathematical model described it the section 2.4 was applied, allowing the evaluation, for each operating condition of the network, of the nodal heads, distributed discharge, mean flow velocities through the links, and so on. Then, the values of local and global Performances Indexes were calculated for each simulation. Once the Performance Indexes were known, it was possible to evaluate the Local and Global Reliability Functions. Some examples of Local Reliability Functions for nodes distributed along the path n.4 of the network are plotted in the Fig. 2. It is possible to observe that, moving from upstream (node 147) to downstream (node 153), the performance of the system is decreasing, since the terrain elevation pattern and the increasing distance from the source (node 1). Thus, given a certain value of the local index (*lpi_ddn* or *lpi_can*), the probability of exceeding that value is always diminishing moving upstream to downstream

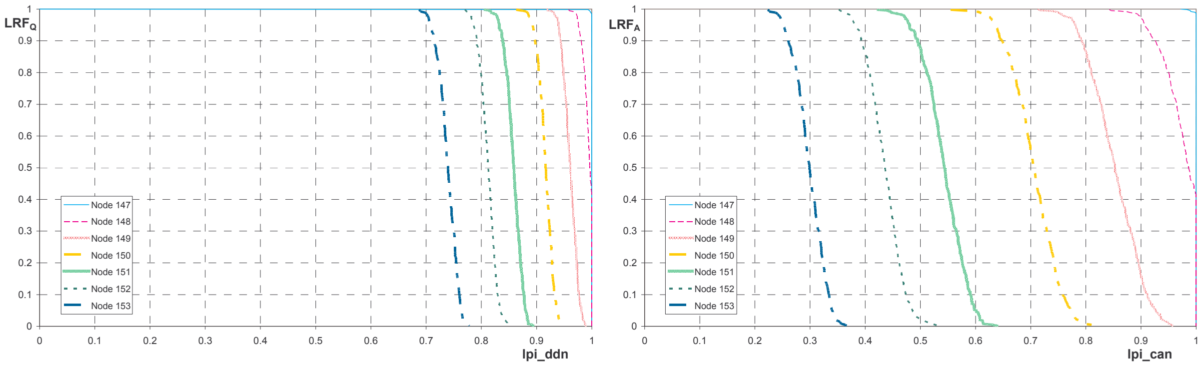

Fig.2. Local Reliability Functions for nodes located along the path n.4

In the Fig. 3 both Global Reliability Functions are shown for the Case 1 (F=0.95%). It is possible to verify that, the case-study network, though designed by correctly using a classical probabilistic approach, is not sufficient to guarantee the users' satisfaction. For instance, the probability to supply less than the 90% of the discharges requested is greater than 90%. On the other hand, the probability to irrigate less than the 70% of the whole area served by the network is greater than 90%. In the same Fig.3, the Global Reliability Functions for the Case 2 (F=0.975%) are also shown. The trends indicate very clearly that the correct design of the network had to be made considering not only a probabilistic approach but also the actual spatial pattern of the terrain elevations and the actual hydraulic behaviour of the irrigation system.

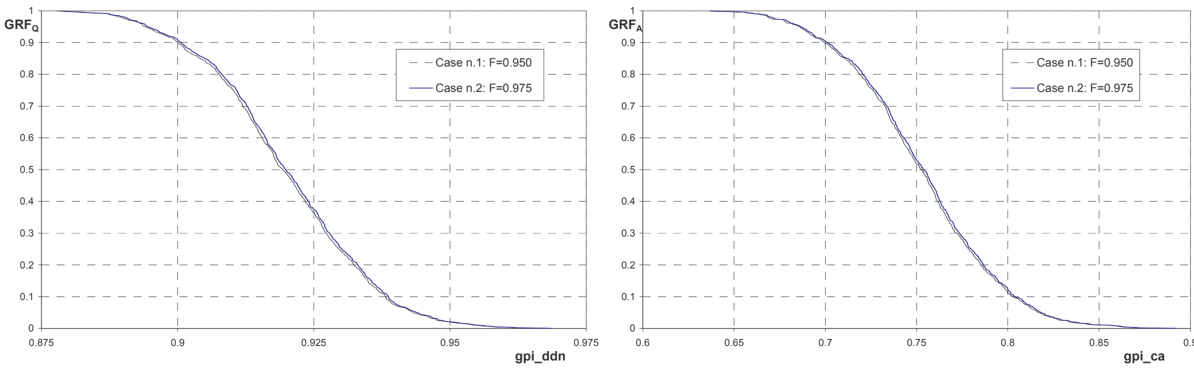

Fig.3. Global Reliability Functions evaluated for the two cases

In the Fig. 4, the same comparison made at global scale in Fig. 4 is performed at local scale, with reference to the node 77 (see Fig.1).

4. CONCLUSIONS

The optimal design of an on-demand pressurized irrigation network, also if performed by means of a classical probabilistic approach, doesn't seem able to guarantee, in all circumstances, the reliability of the system with reference to the variability of users'demand. In this paper, a new approach for the evaluation of the reliability of irrigation networks is then proposed: this approach seems more effective than existing ones [8] [10] in evaluating the performance of this type of irrigation systems. In order to perform the evaluation of the capability of the system to satisfy the users' demand, a complete hydraulic model for the network simulation is coupled with the application of a Monte Carlo technique. This approach allows the evaluation of Reliability Functions, after calculating the above defined Performance Indexes, which physical meaning is clear. The approach proposed was applied to a medium size pressurized irrigation network, similar to those that can be frequently encountered in the applications. The time elapsed to perform the complete set of simulations was of 8 hours, using a Pentium IV 3.2, with 1 Gb of 400 DIMM RAM memory, and it is fully compatible with the current technical practice.

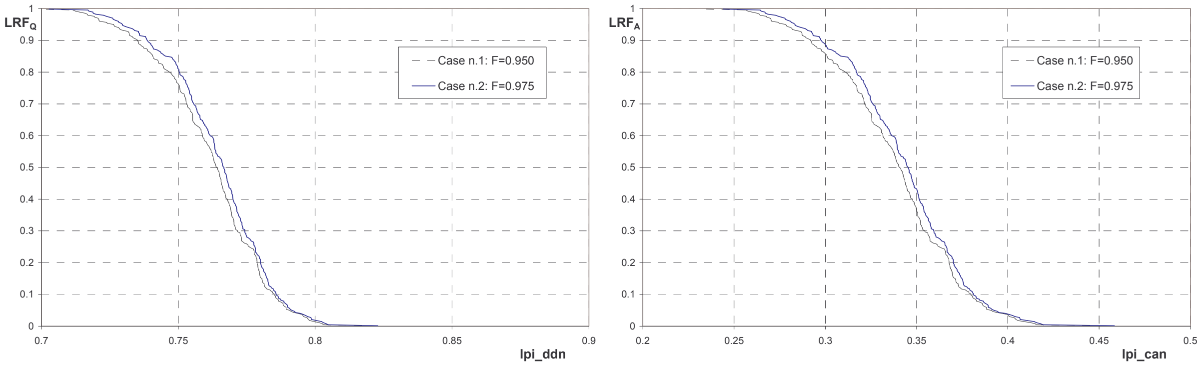

Fig.4. Local Reliability Functions evaluated, at node 77, for the two cases

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